Concepts for Reduction of Blade/Vortex Interaction Noise

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Based upon analysis of a simplified physical model, the critical parameters controlling noise generation by blade/vortex interaction in modern helicopters are identified and their sensitivity evaluated. In particular, the noise production is shown to depend upon the incoming vortex strength, the lift on the blade at the time of interaction, the span over which essentially two dimensional interaction accurs, and the miss distance of vortex. Knowledge of these parameters, which are partially under the control of the helicopter designer, suggests several concepts for reduction of this noise source.

Nomenclature

a_0	= ambient speed of sound
c	= airfoil chord
ℓ	= length over which two-dimensional interaction occurs
L	= lift per unit length of airfoil
$\boldsymbol{M}_0$	= sound speed vector
m_0	= Mach number, = magnitude of sound speed vector
$p(x,t)$	= acoustic pressure
(r,θ)	= blade-fixed polar coordinate system
s	= span of blade
t	= time
t^*	= $t+x/a_0$, = observer time
u,v	= velocity components in blade-fixed coordinate system
U_0	= incoming flow speed
$\boldsymbol{v}$	= flow velocity vector, = $[u,v,0]$
V	= helicopter forward speed
$\boldsymbol{x}=(x_1,x_2,x_3)$	= observer position in blade-fixed coordinate system
x	= magnitude of vector $\boldsymbol{x}$, = observer distance
(y_1,y_2,y_3)	= blade-fixed rectangular coordinate system
α	= angle of attack of helicopter blade
Γ	= circulation of incoming vortex
Γ_0	= circulation around airfoil
ρ	= radial coordinate from rotor hub
ρ_0	= ambient density
$\boldsymbol{\phi}$	= vector of potentials, = $[\phi_1,\phi_2,y_3]$
ϕ_1,ϕ_2	= velocity potentials
$\boldsymbol{\omega}$	= flow vorticity vector, = $[0,0,\omega]$
$\omega(y_1,y_2)$	= vorticity distribution
Ω	= angular velocity of rotor
$(\dot{\ })$	= time derivative
$(\bar{\ })$	= spatial average

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Introduction

OVER the past few years, the authors have studied the blade/vortex interaction problem in two dimensions using potential flow,[1] incompressible Navier-Stokes,[2] and Euler[3] codes to compute the fluid dynamics. Out of these studies has come an appreciation for the complexity of this interaction and an awareness of the improbability of precise prediction of the resulting noise radiation for the case of a helicopter in flight. For example, the acoustic emission has been shown[1] to depend critically upon the vortex path that, for the case of an actual helicopter, depends upon its speed and flight attitude as well as the interaction of the tip vortices from all the blades, the boundary condition imposed by the fuselage, the turbulence of the atmosphere, and other factors such as the lift time history of each blade. Thus, the outlook for precise prediction appears remote.

These same studies have, however, produced an increased understanding of the physics of the interaction process that leads to the noise radiation. Thus, it is perhaps not overly optimistic to inquire whether certain "rules of thumb" might be developed that would guide the designer in reduction of this source of helicopter noise. This paper will examine that question.

It is useful to first review what is known about blade/vortex interaction (BVI) noise. As the helicopter blades rotate, vorticity is continuously shed from the tip of each blade, produced by a flow around the tip from the (higher-pressure) lower surface to the (lower-pressure) upper surface of the lifting blade. This flow is necessary to relieve the pressure difference at the end of the blade. The vorticity quite rapidly rolls up into what are known as tip vortices, one trailing from the tip of each blade as shown in Fig. 1. Once formed, these vortices tend to move downward away from the rotor plane, due to the downflow through the rotor, and thus ordinarily do not interact strongly with following blades of the rotor. However, during certain maneuvers, the helicopter can actually fly into its own wake, such that close encounters between the tip vortices and the rotor blades occur. The acceleration of the vortices resulting from these encounters causes the blade/vortex interaction sound to be radiated. This phenomenon generally tends to occur for only a very small portion of a typical helicopter flight profile, when the helicopter is in forward motion and descending[4] at a moderate rate as shown in Fig. 2. Unfortunately, this small portion usually finds the helicopter at low altitude, often over populated areas. In addition, the frequency content introduced by this source is in the range to which human beings are extremely sensitive.

Second, BVI tends to occur over only a very small portion of the rotor disk, on the side of advancing blades. Figure 3 depicts schematically the positions of the tip vortices shed by either a two- or four-bladed rotor of a helicopter in forward flight with velocity V. In the two-bladed case shown in Fig. 3a, note that the advancing rotor blade can interact with the tip vortex shed by the previous blade (180 deg ahead). A similar phenomenon can also occur on the advancing side of a four-bladed rotor. In such a region, the blade can experience an interaction with the tip vortex of the blade either 180 or 270 deg ahead, as shown in Fig. 3b. As can be seen in this figure, other interactions are possible. For example, near the top of the figure, the blade is crossing over the vortex shed from the blade 90 deg ahead. However, in this case, the vortex is nearly perpendicular to the blade as shown in Fig. 4a. Experimental results[5] indicate that the more nearly parallel the tip vortex is to the blade at the time of interaction (as shown in Fig. 4b), the greater the noise radiation. Thus, a perpendicular interaction results in little noise. On the retreating side of the rotor, it can further be seen from Fig. 3b that a more parallel interaction is possible. However, on the retreating side, the direction of rotation of the vortex is opposite to the direction of rotation of the flow about the airfoil, as opposed to the advancing side where the directions are the same. Numerical results[2] indicate that the interaction is stronger, producing more noise radiation, in the latter case. This is confirmed by extensive wind tunnel experimental source location studies[6] on a four-bladed rotor that have determined the primary sources of BVI to be located between 70 and 80% of the blade span and at an angle between 70 and 80 deg on the advancing side of the rotor disk as shown in Fig. 5. More recent work[7] on a two-bladed rotor has indicated a similar result, although the angle of the blade at the time of interaction is closer to 90 deg. From Fig. 3a, it is apparent that the angle at which the interaction takes place will be larger for a two-bladed rotor. These observations will be utilized in developing a simplified model of the blade/vortex interaction phenomenon.

Analysis

Since the blade/vortex interaction takes place well inboard of the tip and is maximized when the vortex is parallel to the blade, a two-dimensional model such as shown in Fig. 6 should be adequate. Here, a blade-fixed coordinate system is utilized in which the airfoil section is at an angle of attack α with respect to the incoming flow U_0, which produces a lift on the airfoil represented by the circulation Γ_0. The incoming flow U_0 includes both the rectilinear component V due to the forward motion of the helicopter and the rotational component $\rho\Omega$, where Ω is the angular velocity of the rotor. The previously mentioned measurements indicate $\rho \simeq 0.75s$, where s is the span of the blade, i.e.,

$$U_0 \simeq V + 0.75s\,\Omega \tag{1}$$

This velocity will be assumed subsonic, such that characteristic compressibility effects (i.e., shocks) will not be present. This condition is often valid in the region of interest even when the blade tip speed is sonic. At times, the addition of the vortex-induced velocity to the blade flowfield can apparently cause sonic conditions on the blade. However, that case will not be considered here. The incoming vortex is represented by the vorticity distribution $\omega = \omega(y_1, y_2)$ shown in Fig. 6.

Consider a length ℓ of the airfoil over which the two-dimensional interaction takes place. From the previously cited data, $\ell \simeq 0.1s$. The origin of coordinates is placed at the midpoint of this length and the observer is at the farfield position $\boldsymbol{x} = (x_1, x_2, x_3)$ such that $|\boldsymbol{x}| \gg \ell$. Assume further that the characteristic wavelength of the sound radiation is much larger than the airfoil chord and that the sound sources are within a wavelength of the airfoil. The validity of these assumptions for this geometry was substantiated by the results of Hardin and Mason[1] and Hardin and Lamkin.[3] Utilizing the theory of aerodynamic sound generation[8] and the low-frequency Green's function approach developed by Howe,[9] the time history of the acoustic pressure radiated to the farfield observer position $\boldsymbol{x}$ is given by

$$p(\boldsymbol{x},t) \simeq -\frac{\rho_0}{4\pi a_0 x[1+\boldsymbol{M}_0\cdot(\boldsymbol{x}/x)]}\frac{\partial}{\partial t} \times \int_{\text{vol}} \left\{(\boldsymbol{\omega}\times\boldsymbol{v})\cdot\nabla\left[\left(\frac{\boldsymbol{x}}{x}-\boldsymbol{M}_0\right)\cdot\boldsymbol{\phi}\right]\right\}\mathrm{d}\boldsymbol{y} \tag{2}$$

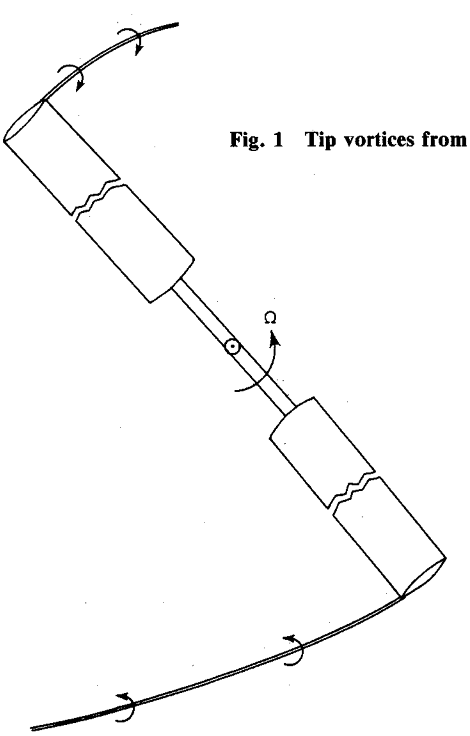

Fig. 1 Tip vortices from lifting rotor blades.

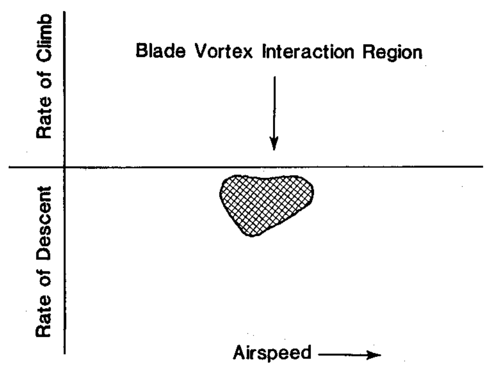

Fig. 2 Flight regime for occurrence of BVI.

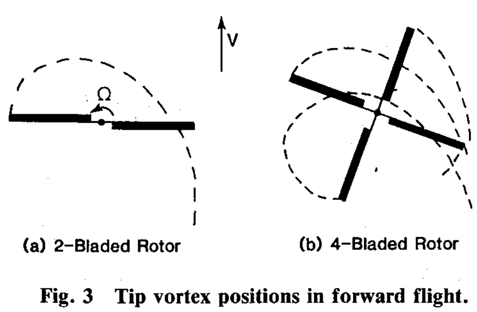

Fig. 3 Tip vortex positions in forward flight.

where p is the acoustic pressure at the observer location at time t, ρ_0 and a_0 are the ambient density and speed of sound in the flow, respectively, $x = |\boldsymbol{x}|$, $\boldsymbol{M}_0 = (M_0 \cos\alpha, M_0 \sin\alpha, 0)$ where $M_0 = U_0/a_0$ and $\boldsymbol{\omega}$ and $\boldsymbol{v}$ are the flow vorticity and velocity vectors, respectively. The integral is over the total volume of the flow and the integrand is evaluated at the retarded time $t-(x-\boldsymbol{M}_0\cdot\boldsymbol{x})/a_0$. The vector $\boldsymbol{\phi}$ contains potentials such that ϕ_i is the potential of incompressible flow about the body in which the flow at large distances from the body is of unit speed in the i direction. Thus, $\boldsymbol{\phi} = (\phi_1, \phi_2, y_3)$.

Now, in this two-dimensional flow, $\boldsymbol{\omega} = (0, 0, \omega)$ and $\boldsymbol{v} = (u, v, 0)$. Further, the Mach number dependence may be neglected as a second-order effect. Thus, Eq. (2) becomes

$$p(x,t^*) \simeq -\frac{\rho_0 \ell}{4\pi a_0 x}\frac{\partial}{\partial t}\left[\frac{x_1}{x}\int \mathrm{d}y_1 \int \mathrm{d}y_2 \omega \left(\underset{(a)}{u\frac{\partial \phi_1}{\partial y_2}} - \underset{(b)}{v\frac{\partial \phi_1}{\partial y_1}}\right) + \frac{x_2}{x}\int \mathrm{d}y_1 \int \mathrm{d}y_2 \omega \left(\underset{(c)}{u\frac{\partial \phi_2}{\partial y_2}} - \underset{(d)}{v\frac{\partial \phi_2}{\partial y_1}}\right)\right] \tag{3}$$

where $t^* = t + x/a_0$ is the reception time.

In order to simplify this expression, note that $u \gg v$. Further, by definition of the potentials,

$$\frac{\partial \phi_1}{\partial y_1} \simeq \frac{\partial \phi_2}{\partial y_2} \simeq 1 \gg \frac{\partial \phi_1}{\partial y_2} \simeq \frac{\partial \phi_2}{\partial y_1}$$

Hence, term c will produce the largest integral in Eq. (3). A numerical computation[2] found the nondimensional integrals of terms a-d to be 0.005, 0.022, 0.37, and 0.009, respectively, at a time when the vortex was strongly interacting with the airfoil. In addition, the time rate of change of the integral of term c is the largest. Thus, Eq. (3) is well approximated by

$$p(x,t^*) \simeq \frac{-\rho_0 \ell x_2}{4\pi a_0 x^2}\frac{\partial}{\partial t}\int \mathrm{d}y_1 \int \mathrm{d}y_2 \omega u \tag{4}$$

Finally, note that contributions to the integral of Eq. (4) occur only where both the velocity u and vorticity ω are nonzero. Thus, if the boundary layer and wake of the airfoil are neglected as being unimportant to the BVI problem, the integral of Eq. (4) will encompass only the incoming vortex. Further, if the distortion of the vortex due to the interaction is negligible (i.e., assuming a near miss rather than a collision), the vortex will move as a rigid body with space-averaged velocity $\bar{u}$. Then, Eq. (4) may be further approximated by the simple expression

$$p(x,t^*) \simeq \frac{-\rho_0 \ell x_2}{4\pi a_0 x^2}\Gamma \dot{\bar{u}} \tag{5}$$

where

$$\Gamma = \int \mathrm{d}y_1 \int \mathrm{d}y_2 \omega$$

is the total circulation of the incoming vortex that is conserved except for slow viscous effects.

Equation (5) is possibly the simplest expression retaining the essential physics of the BVI noise production process and illustrating the fundamental dependence of the radiated noise on the vortex acceleration $\dot{\bar{u}}$. A "back-of-the-envelope" type of evaluation of this quantity may be obtained by considering the geometry shown in Fig. 7. Here, both the airfoil and the

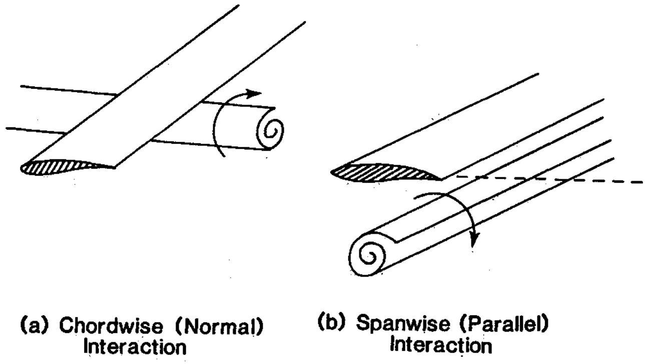

Fig. 4 Types of blade/vortex interaction.

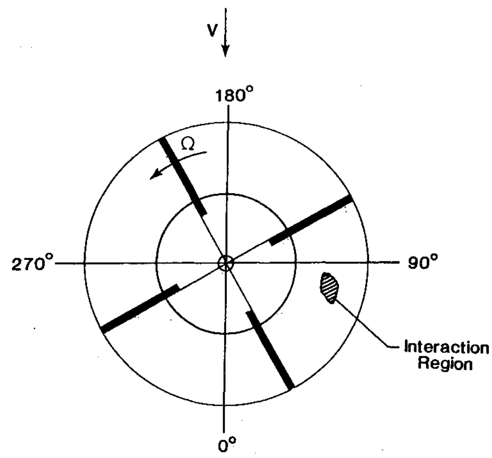

Fig. 5 Blade/vortex interaction region in rotor disk as defined by source location.

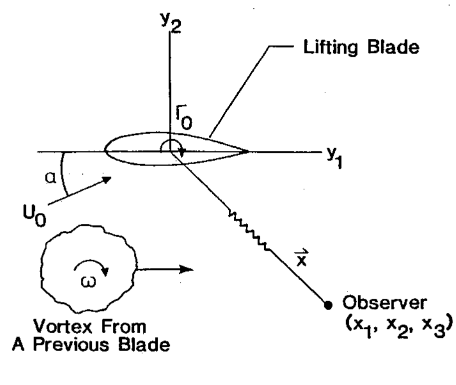

Fig. 6 Two-dimensional BVI geometry in blade-fixed coordinate system.

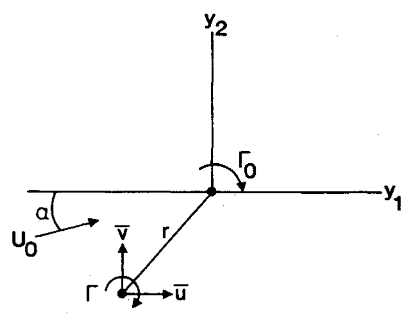

Fig. 7 Simplified BVI model geometry.

incoming vorticity distribution have been replaced by concentrated vortex filaments with the airfoil vortex considered to be bound. The directions of rotation of both the flow around the airfoil and the incoming vortex are clockwise. Thus, by convention, both the circulation of the airfoil and that of the vortex are negative in the region of interest. The velocity components of the incoming vortex are then given by

$$\bar{u} = U_0 \cos\alpha - \frac{\Gamma_0}{2\pi r}\sin\theta \tag{6}$$

$$\bar{u} = U_0 \sin\alpha + \frac{\Gamma_0}{2\pi r}\cos\theta \tag{7}$$

in polar coordinates.

Now, from Eq. (5), the noise production depends upon the acceleration of the vortex in the streamwise direction. Thus, differentiating Eq. (6), the acceleration of the vortex in the streamwise direction may be shown to be

$$\dot{\bar{u}} = \frac{\Gamma_0}{2\pi r^2}\left[U_0 \sin(2\theta-\alpha) - \frac{\Gamma_0}{2\pi r}\cos\theta\right] \tag{8}$$

and, employing this relation in Eq. (5), the final expression for the acoustic pressure time history produced by the BVI phenomenon becomes

$$p(x,t^*) \simeq \underbrace{\left(\frac{-\rho_0 x_2}{8\pi^2 a_0 x^2}\right)}_{\text{I}} \underbrace{\left(\frac{\Gamma\Gamma_0 U_0 \ell}{r^2}\right)}_{\text{II}} \underbrace{\left[\sin(2\theta-\alpha) - \frac{\Gamma_0}{2\pi U_0 r}\cos\theta\right]}_{\text{III}} \tag{9}$$

where $r(t)$ and $\theta(t)$ depend upon the vortex trajectory obtained by the numerical solution of Eqs. (6) and (7).

The above equations may be nondimensionalized utilizing Γ_0/U_0 and Γ_0/U_0^2 as the length and time scales, respectively. Figure 8 depicts a typical trajectory of the incoming vortex obtained from numerical solution of Eqs. (6) and (7) with the initial condition $(y_1', y_2') = (-5.0, -0.7)$ and an angle of attack of 5 deg. In this case where the vortex passes below the airfoil, note that the airfoil lift attracts the vortex vertically as it approaches and then repels it after it passes. The vortex is also alternately decelerated and then accelerated in the streamwise direction. It is this respect of the interaction that leads to the primary noise radiation.

The normalized acoustic pressure time history, $p' = p \div \rho_0 U_0^2$, in this case is shown in Fig. 9. This typical interaction noise signature is calculated from Eq. (9) utilizing the values of $r(t)$ and $\theta(t)$ obtained from the previous computation. Note that the signature, again for vortex passage below the airfoil and for an observer position below the rotor plane, consists of a positive peak followed by a somewhat larger negative peak. This prediction is in reasonably good agreement with experimental data[4,10,11] over a wide range of rotor tip speeds, although the data often show the positive peak to be the larger of the two.

Further insight into the noise generation process can be obtained by analysis of Eq. (9). For the observer below the rotor plane (i.e., $x_2 < 0$) and the signs of Γ and Γ_0 both negative, as is true in the region of interest, the sign of the quantity multiplying term III positive. Further, note that the second quantity in term III is negligible for most of the vortex trajectory since, for a Joukowski airfoil,[12]

$$\Gamma_0 = \pi c U_0 \sin\alpha \tag{10}$$

where c is the chord of the airfoil, while for other airfoils this value is generally reduced due to separation. Thus, taking as a typical "miss distance," $r = c/2$,

$$\left|\frac{\Gamma_0}{2\pi U_0 r}\right| < \sin\alpha$$

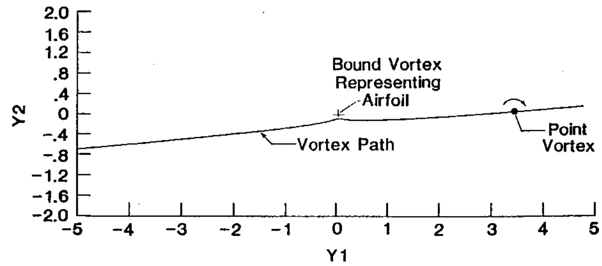

Fig. 8 Typical vortex trajectory.

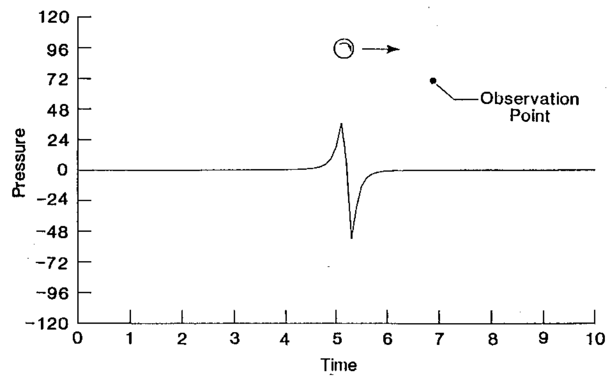

Fig. 9 Typical acoustic pressure time history.

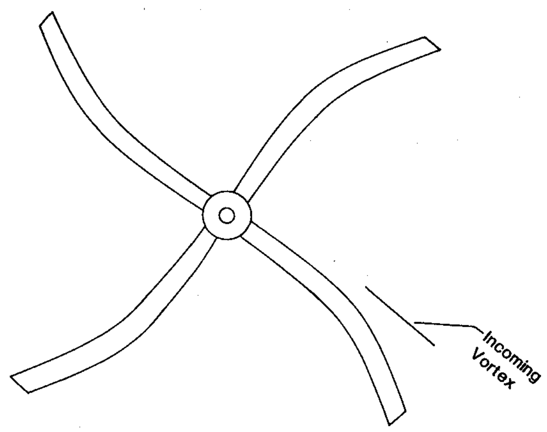

Fig. 10 Rotor designed to reduce interaction length.

and the second quantity in term III Eq. (9) is generally seen to be negligible. Only for a very close interaction will this term be of importance. The result of this analysis is that the sign of the acoustic pressure is governed by the sign of the term $\sin(2\theta-\alpha)$, which has the property that

$$\sin[\sin(2\theta-\alpha)] = +\frac{\alpha}{2} < \theta < \frac{\pi}{2} + \frac{\alpha}{2}$$

$$= -\frac{\pi}{2} + \frac{\alpha}{2} < \theta < \pi + \frac{\alpha}{2}$$

$$= +\pi + \frac{\alpha}{2} < \theta < +\frac{3\pi}{2} + \frac{\alpha}{2}$$

$$= -\frac{3\pi}{2} + \frac{\alpha}{2} < \theta < 2\pi + \frac{\alpha}{2}$$

Thus, as seen by an observer below the rotor plane, a vortex passing above the airfoil will produce a negative peak, as it is

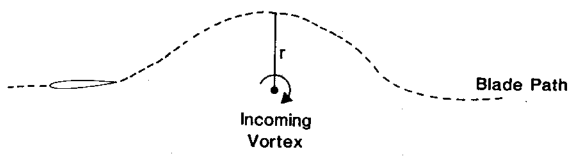

Fig. 11 Blade path modification to increase miss distance.

accelerated by the lift on the airfoil, followed by a positive peak as it decelerates, while a vortex passing below will result in the opposite time history. Which of the two will be more intense depends upon the miss distance r. However, vortices passing below the airfoil are attracted toward it, as shown in Fig. 8, thus reducing their miss distance; while vortices passing above are repelled, thus increasing their miss distance.

Concepts for BVI Noise Reduction

Interpretation of Eq. (9) yields several interesting possibilities for noise reduction. Note that the quantities in term I depend upon the observer position and fundamental physical parameters of the media over which there can be no control. Further, the quantities in term III depend primarily upon the vortex trajectory, which is difficult to predict due to the reasons cited earlier. Therefore, it is only the parameters in term II over which the designer might hope to exert control. This term may be written in an even more revealing form by noting that the lift per unit length on the blade is given by

$$L = \rho_0 u_0 \Gamma_0 \tag{11}$$

Thus,

$$\text{Term II} = \left(\frac{\Gamma\Gamma_0 U_0 \ell}{r^2}\right) = \frac{\Gamma L \ell}{\rho_0 r^2} \tag{12}$$

which reveals the fundamental importance of the blade lift to the BVI noise generation process. The importance of blade loading to the helicopter noise problem in general has been known for some time.[13] However, its role in BVI noise generation has not been fully appreciated. In the following, the parameters in term II will be considered individually.

Incoming Vortex Strength Γ

The first parameter in Eq. (12), the incoming vortex circulation Γ, has been the subject of attempts at modification in the past, primarily by changes in the blade tip shape or by the introduction of winglets—all without dramatic success. In some sense, one is fighting fundamental physical laws in attempting such control because a lifting airfoil must produce a tip vortex. However, recall that the tip vortex is shed continuously from the blade and that its strength is governed by the lift on the blade. Thus, a modification of the lift time history of the blade at the time the incoming vortex was shed from the blade might be another way of achieving the same objective. One way of achieving such control would be to utilize the higher harmonic control (HHC) systems that are already being developed to enhance the vibration characteristics of helicopters.[13] Such systems allow control of the blade angle of attack, and thus its lift, at any azimuth angle throughout the blade rotation.

Blade Lift L

The second parameter in Eq. (12), the lift per unit length of the blade at the time of interaction, perhaps offers a better means of control. This could be reduced by the standard techniques of increasing the number of blades, using longer blades or reducing the weight of the helicopter. Reducing the weight would also have the virtue of reducing the incoming vortex strength Γ, while increasing the number of blades might increase the number of interactions—thus yielding little advantage. Or, since BVI occurs over only a very small portion of the rotor disk plane, as shown in Fig. 5, a reduction in the blade lift over only this small portion might result in a substantial reduction in the noise level (6 dB per factor of two lift reduction) in addition to a probable improvement in the vibration characteristics of the helicopter. Recall as well that this lift reduction need occur only during that small portion of the helicopter flight profile for which BVI is a problem, as shown in Fig. 1. Again, one might imagine utilizing an HHC system for this purpose.

Interaction Length ℓ

The third parameter ℓ is the portion of the span over which essentially two-dimensional (parallel) interaction takes place. This length could be modified by adding curvature to the blades. An example of this concept is shown in Fig. 10, although the blades can curve either forward or aft. Optimization of this concept would require experimentation, since curving the blades would change the location from which the tip vortices are shed and thus change the region of interaction. The real question here is for the helicopter designer: considering the centrifugal forces and the delicate stability of a helicopter, would it be possible to build a helicopter with curved blades?

Miss Distance r

Finally, the last parameter r, the separation distance of the vortex from the airfoil, is the reason why BVI is a problem only under descent conditions such as shown in Fig. 1. Would it be possible to raise the blade, through use of the swash-plate mechanism or a hydraulic actuator, as it enters the region where BVI is likely to occur? This concept is shown in Fig. 11 and would need to be done slowly in order to avoid generating additional sound through the blade motion. An active control system where an accelerometer mounted on the blade is used to sense the occurrence of BVI and then raise the next blade would be a possibility. Again, one can imagine many control problems associated with such an attempt. However, a doubling of the miss distance would reduce the noise level by 12 dB.

Conclusions

By a simple analysis, noise production by blade/vortex interaction has been shown to depend upon four parameters: the incoming vortex strength, the lift on the blade, the length over which two-dimensional interaction occurs, and the miss distance of the interaction over which the helicopter designer might exert control. Suggested means for implementing these noise reduction concepts are included. It is recognized that these suggestions of an aeroacoustician will cause many difficulties for the helicopter aerodynamicist and structural dynamicist. However, it appears that real progress toward reduction of blade/vortex interaction noise in the actual flight environment is possible through engineering and experimentation.

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